# Statistics

# Spring 2023

# Lecture 6

Feb 19-8:47 AM

Class QZ 1

| $x$ | $y$ |
|---|---|
| 5 | 13 |
| 6 | 18 |
| 8 | 18 |
| 8 | 20 |
| 10 | 25 |

$n=5$

$x \to L1$, $y \to L2$

$\boxed{\text{STAT}}$ $\boxed{\to}$ CALC

$\boxed{\text{8: LinReg(a+bx)}}$

Find

$a = 3.5$

$b = 2.1$

} Round to 1-decimal

$r^2 = 87\%$ } Round to whole %

$r = .931$ } Round to 3-decimals

Xlist: L1

Ylist: L2

$\boxed{\text{Clear}}$

Calculate

L1, L2

$\boxed{\text{enter}}$

Mar 7-9:01 PM

Addition Rule $\boxed{SG\ 11}$

Keyword : OR

Single Action event

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

ex: $P(A) = .6$ , $P(B) = .7$, $P(A \text{ and } B) = .5$

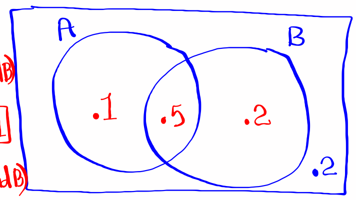
$P(\bar{A}) = 1 - P(A) = \boxed{.4}$

$P(\bar{B}) = 1 - P(B) = \boxed{.3}$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$= .6 + .7 - .5 = \boxed{.8}$

Venn Diagram




$P(A \text{ only}) = P(A) - P(A \text{ and } B)$

$= .6 - .5 = \boxed{.1}$

$P(B \text{ only}) = P(B) - P(A \text{ and } B)$

$= .7 - .5 = \boxed{.2}$

Total Prob = 1 ✓

Mar 14-6:53 PM

$P(HB) = .65$

$P(FF) = .55$

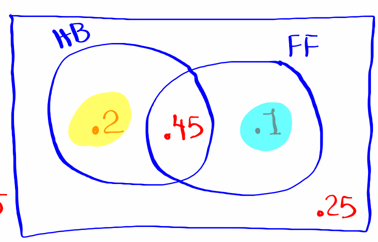
$P(HB \text{ and } FF) = .45$

$P(\overline{HB}) = 1 - .65 = \boxed{.35}$

$P(HB \text{ or } FF) = P(HB) + P(FF) - P(HB \text{ and } FF)$

$= .65 + .55 - .45 = \boxed{.75}$

Venn Diagram




$P(HB \text{ only}) = .65 - .45$

$= .2$

$P(FF \text{ only}) = .55 - .45$

$= .1$

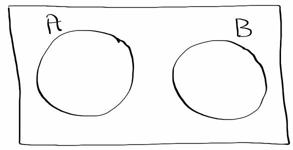
Total = 1

$P(HB \text{ only} \text{ OR } FF \text{ only}) = .2 + .1 = \boxed{.3}$

Mar 14-6:59 PM

Two events are Mutually Exclusive Events / Disjointed Events if they do not happen together.

A & B are M.E.E. $\Leftrightarrow$ $P(A \text{ and } B) = 0$

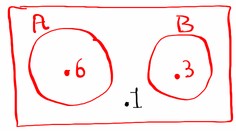



ex: $P(A) = .6$, $P(B) = .3$, A & B are M.E.E.

$P(\overline{A}) = 1 - .6 = \boxed{.4}$

$P(\overline{B}) = 1 - .3 = \boxed{.7}$

$P(\underbrace{A \text{ and } B}_{\text{Disjointed Events}}) = 0$




Total = 1

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$= .6 + .3 - 0 = \boxed{.9}$

$P(\overline{A \text{ or } B}) = 1 - P(A \text{ or } B) = 1 - .9 = \boxed{.1}$

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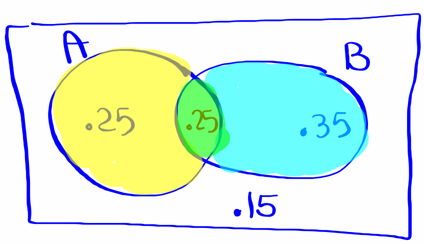
Complete the Venn Diagram below




$P(A \text{ only}) = .25$

$P(B \text{ only}) = .35$

$P(A \text{ and } B) = .25$

Total = 1

$P(A) = .25 + .25 = \boxed{.5}$, $P(B) = .35 + .25 = \boxed{.6}$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$= .5 + .6 - .25 = \boxed{.85}$

$P(\underbrace{A \text{ or } B \text{ but not both}}_{A \text{ only OR } B \text{ only}}) = .25 + .35 = \boxed{.6}$

Mar 14-7:12 PM

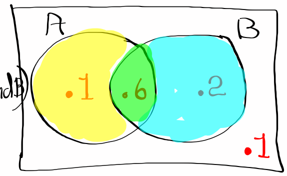
## De Morgan's Law:

$P(\bar{A} \text{ and } \bar{B}) = P(\overline{A \text{ or } B})$

$P(\bar{A} \text{ or } \bar{B}) = P(\overline{A \text{ and } B})$

Given $P(A) = .7$, $P(B) = .8$, $P(A \text{ and } B) = .6$

1) Make Venn Diagram




2) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$= .7 + .8 - .6$

$= \boxed{.9}$

Total = 1

3) $P(\bar{A} \text{ and } \bar{B}) = P(\overline{A \text{ or } B}) = 1 - P(A \text{ or } B)$

De Morgan's Law

$= 1 - .9 = \boxed{.1}$

4) $P(\bar{A} \text{ OR } \bar{B}) = P(\overline{A \text{ and } B}) = 1 - P(A \text{ and } B)$

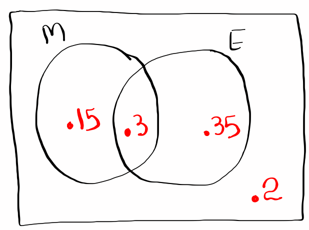
$= 1 - .6 = \boxed{.4}$

Mar 14-7:19 PM

$P(\text{Math}) = .45$

$P(\text{English}) = .65$

$P(\text{Math and English}) = .3$




$P(\text{Math only}) = .45 - .3 = .15$

Total = 1

$P(\text{English only}) = .65 - .3 = .35$

$P(\text{Math or English}) = P(M) + P(E) - P(M \text{ and } E) = \boxed{.8}$

$P(\text{Math, English, not both}) = .15 + .35 = \boxed{.5}$

Use DeMorgan's Law to find

$P(\bar{M} \text{ and } \bar{E}) = P(\overline{M \text{ or } E}) = 1 - .8 = \boxed{.2}$

$P(\bar{M} \text{ OR } \bar{E}) = P(\overline{M \text{ and } E}) = 1 - .3 = \boxed{.7}$

SG 11

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odds vs. Probability (SG 12)

Notation :

odds in favor of event E are $a : b$

odds against event E are $b : a$

I flipped a coin 100 times, it landed tails 70 times.

odds in favor of landing tails

$\underset{\text{Tails}}{70} : \underset{\overline{\text{Tails}}}{30} \Rightarrow \boxed{7:3}$

odds against landing tails $\Rightarrow \boxed{3:7}$

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odds in favor of event E

$\underset{\text{happens}}{\# E} : \underset{\text{happens}}{\# \overline{E}}$

A standard deck of playing cards has 52 cards ∴ 4 Aces.

$P(\text{Draw Ace}) = \frac{4}{52} = \frac{1}{13}$

odds in favor of drawing Ace

$\# \text{Aces} : \# \overline{\text{Aces}}$

$4 : 48$

Simplify $1:12$

odds against drawing Ace

$12 : 1$

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odds in favor of event E are $a:b$

$P(E) = \frac{a}{a+b}$ , $P(\bar{E}) = \frac{b}{a+b}$

ex: Suppose odds in favor of event E are $4:21$

1) odds against event E $\Rightarrow$ $21:4$

2) $P(E) = \frac{4}{4+21} = \boxed{\frac{4}{25}}$

3) $P(\bar{E}) = \frac{21}{4+21} = \boxed{\frac{21}{25}}$

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Suppose odds in favor of Lakers win the championship is $1:39$.

\$1 bet $\Rightarrow$ \$39 Profit

1) odds against Lakers win. championship.

$\boxed{39:1}$

2) $P(\text{win}) = \frac{1}{1+39} = \boxed{\frac{1}{40}}$

3) $P(\overline{\text{win}}) = \frac{39}{1+39} = \boxed{\frac{39}{40}}$

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How to find odds for event E when $P(E)$ is given.

odds in favor of event E are

$$P(E) : P(\overline{E})$$

ex: Suppose $P(E) = .12$

1) $P(\overline{E}) = 1 - P(E) = \boxed{.88}$

2) odds in $\boxed{\text{favor}}$ of event E.

$P(E) : P(\overline{E})$ → $\boxed{3:22}$

$.12 : .88$

$.12 \boxed{\div} .88$ $\boxed{\text{MATH}}$ $\boxed{\text{1: ▶Frac}}$ $\boxed{\text{Enter}}$

3) odds $\boxed{\text{against}}$ event E. → $\boxed{22:3}$

Mar 14-8:02 PM

$P(\text{passing a driving Test}) = .4$

1) $P(\overline{\text{Pass}}) = 1 - .4 = \boxed{.6}$

2) odds in $\boxed{\text{favor}}$ of passing.

$P(\text{Pass}) : P(\overline{\text{Pass}})$ → $\boxed{2:3}$

$.4 : .6$

3) odds $\boxed{\text{against}}$ Passing. → $\boxed{3:2}$

Mar 14-8:07 PM

Multiplication Rule

Key word AND

Multiple Action Event

$P(A \text{ and } B) = P(A) \cdot P(B|A)$

A happens, then B happens

Given

Consider a fair Coin.

$P(T) = .5$ $P(H) = .5$

Flip it twice

$\underbrace{TT \quad TH \quad HT \quad HH}_{\text{Sample Space}}$

$P(TT) = P(T) \cdot P(T) = (.5)(.5) = \boxed{.25}$

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3 Dimes, 7 Nickels,

Select 2 Coins with replacement

DD DN ND NN

$P(NN) = P(N) \cdot P(N)$

$= \frac{7}{10} \cdot \frac{7}{10} = \frac{49}{100} = \boxed{.49}$

A standard deck of playing Cards has 52 Cards, 12 Faces.

Select 2 Cards without replacement

$FF \quad F\bar{F} \quad \bar{F}F \quad \bar{F}\bar{F}$

$P(FF) = \frac{12}{52} \cdot \frac{11}{51} = \boxed{\frac{11}{221}}$

Let's draw 3 Card, no replacement

$P(FFF) = \frac{12}{52} \cdot \frac{11}{51} \cdot \frac{10}{50} = \boxed{\frac{11}{1105}}$

Mar 14-8:21 PM

**Independent Events:**

one outcome does not change the Prob. of next outcome.

$P(Boy) = .5$ $P(Girl) = .5$

Roll a fair die

$P(\text{land } 6) = \frac{1}{6}$ on every roll.

multiple-choice questions, 4 choices, 1 correct choice.

$P(\text{guess Correct}) = \frac{1}{4}$ on every question

$P(\text{guess } \overline{\text{correct}}) = \frac{3}{4}$ " "

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If A and B are Independent events,

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

Toss a loaded Coin twice

$P(T) = .3$ $P(H) = .7$

TT TH HT HH

$P(TT) = (.3)(.3) = \boxed{.09}$

$P(1T \text{ \& } 1H) = 2 \cdot (.3)(.7) = \boxed{.42}$

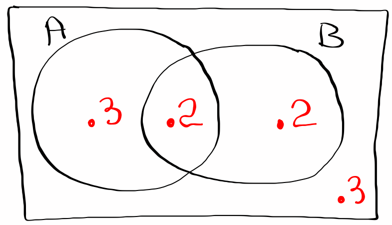
Mar 14-8:33 PM

$P(A) = .5$ , $P(B) = .4$ , A & B are independent events.

$P(A \text{ and } B) = P(A) \cdot P(B)$
$= (.5) \cdot (.4) = \boxed{.2}$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
$= .5 + .4 - .2 = \boxed{.7}$

Make Venn Diagram




Total = 1

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$P(A) = .75$

$P(B) = .8$

A & B are independent events.

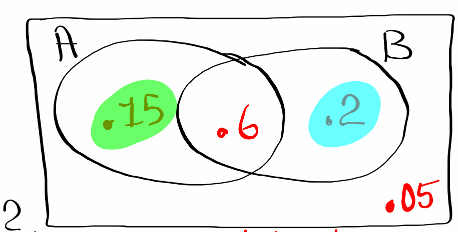
1) $P(\overline{A}) = 1 - P(A) = \boxed{.25}$

2) $P(\overline{B}) = 1 - P(B) = \boxed{.2}$

3) $P(A \text{ and } B) = P(A) \cdot P(B) = \boxed{.6}$

4) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = \boxed{.95}$

5) Make Venn Diagram




6) $P(A \text{ only OR } B \text{ only}) = .15 + .2 = \boxed{.35}$

Total = 1

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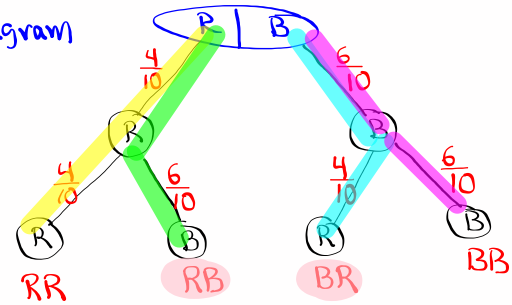
A box has 4 Red & 6 Blue balls

Draw 2 balls with replacement

$P(R) = \frac{4}{10}$ , $P(B) = \frac{6}{10}$

Tree Diagram




$P(RR) = \frac{4}{10} \cdot \frac{4}{10} = \frac{16}{100} = \boxed{.16}$

$P(1R \text{ \& } 1B) = 2 \cdot \frac{4}{10} \cdot \frac{6}{10} = \frac{48}{100} = \boxed{.48}$

$P(BB) = \frac{6}{10} \cdot \frac{6}{10} = \frac{36}{100} = \boxed{.36}$

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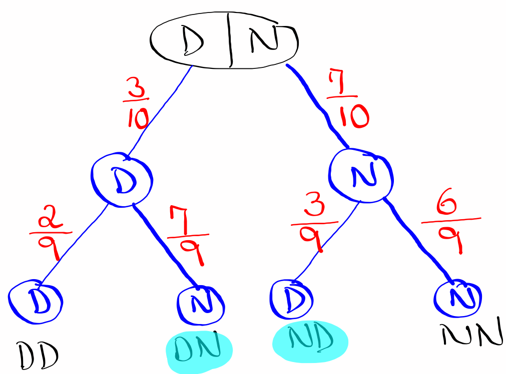
A box has 3 dimes & 7 nickels.

Select 2 Coins, no replacement




$P(DD) = \frac{3}{10} \cdot \frac{2}{9} = \frac{\boxed{6}}{90}$

$P(1D \text{ \& } 1N) = 2 \cdot \frac{3}{10} \cdot \frac{7}{9} = \frac{\boxed{42}}{90}$

$P(NN) = \frac{7}{10} \cdot \frac{6}{9} = \frac{\boxed{42}}{90}$

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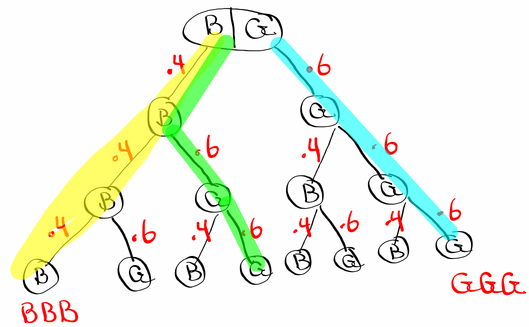
Suppose $P(\text{Boy}) = .4$ $\quad P(\text{Girl}) = .6$

Consider 3 new born babies




$P(BBB) = (.4)(.4)(.4) = \boxed{.064}$

$P(GGG) = (.6)(.6)(.6) = \boxed{.216}$

$P(1B \;\&\; 2G) = 3 \cdot (.4)(.6)(.6) = \boxed{.432}$

BGG
GBG
GGB

$P(2B \;\&\; 1G) = 3 \cdot (.4)(.4)(.6) = \boxed{.288}$

BBG
BGB
GBB

Mar 14-9:02 PM

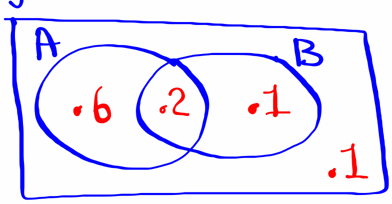
# Class QZ 2

Suppose $P(A) = .8$ , $P(B) = .3$ , $P(A \text{ and } B) = .2$

1) $P(\overline{B}) = 1 - P(B) = \boxed{.7}$

2) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = \boxed{.9}$

3) Construct Venn Diagram.

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